

# Locally Optimized SSAC: LOSSAC

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**Abstract:** A new method of locally optimized sample consensus based on S-estimator has been introduced in this paper (LOSSAC). Locally optimized RANSAC (LORANSAC) and other locally optimized procedure are developed based on M estimator. The weakness of M estimation is the lack of consideration of distribution of data and it is not a function of overall data, because it uses median as the weight value. The proposed method uses the residuals standard deviation to overcome the weaknesses of median. The performance of the improved SSAC is studied and compared with LORANSAC and LOMSAC. The LOSSAC provides reliable results and more efficiency by comparing with the number of inliers estimated under various procedures.

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## 1. Introduction

Robust statistics play vital role in computer tasks, especially for estimating the model parameters computed from a data containing a significant proportion of outliers. RANSAC has been applied in many of the computer vision tasks such as image segmentation and reconstruction, stereo matching, motion segmentation, robust Eigen image matching and elsewhere. In classical formulation of RANSAC, the problem is to find inliers and also it is not known a priori. The RANSAC procedure finds, with a certain probability, that model repeatedly drawing random samples and they almost no assumptions are made about the data. However, it has been observed that RANSAC experiments run much longer.

The local optimization (LO) need additional computation cost. But the embedded iteration step will only be invoked when obtained model is currently optimal which tends to improve the consensus score more rapidly. Obviously, the times of activating LO follows as a logarithmic increase of iteration times, which means local optimization would not influence the speed of the whole algorithm much.

In LO-RANSAC assumptions rarely holds in practice and it makes the above mentioned assumption valid by applying local optimization to the solution estimated from the random sample. In this paper locally optimized sample consensus based on S-estimator procedure (LO-SSAC) has been introduced. LO-SSAC expresses the fact that the model hypothesis based on residual standard deviations. This approach not only increases the numbers of inliers found and also speeds up the entire process of SSAC. The proposed locally optimized method guaranteed the number of samples for which it is insignificant.

The structure of the paper is as follows. Section 2 presents the detailed discussion of Locally Optimization and section 3 presents in Locally Optimized SAC. The proposed procedure LO-SSAC is discussed in section 4. All methods are experimentally tested and evaluated through simulation. The obtained results are shown and discussed in section 5. The paper is concluded in section 6.

## 2. Locally Optimization

The core idea of local optimization is to use an extended two-level iteration frame work. When optional model is obtained, according to these model parameters, we can get more satisfying results through an embedded iterations step which accelerates convergence speed of the algorithm and enhances efficiency. Here optimal means maximum likelihood. Because the number of data points consistent with a model from a randomly selected sample is a random variable with an unknown probability density function, and this density function is the same for all samples, the probability that the  $k^{\text{th}}$  sample will be currently optimal is  $1/k$ . therefore, the expected times that the  $k^{\text{th}}$  sample reaches optimal will be

$$\sum_i^k \frac{1}{x} \leq \int_1^k \frac{1}{x} dx + 1 = \log k + 1 \quad (2.1)$$

The LO step is carried out only if a new maximum in the number of inliers is reached, i.e. when standard RANSAC stores it's so-far-the-best result. How often does this happen. The number of data points consistent with a model from a randomly selected sample can be thought of as a random variable with an unknown density function. This density function is the same for all samples, so the probability that  $k^{\text{th}}$  sample will be the best so far is  $1/k$ . The logarithmic growth of the number of LO step invocations as a function of the number of hypothesize-and-verify cycles allows application of relatively computationally expensive optimization methods without an impact on the overall speed of the algorithm.

Different methods of the best model optimization with respect to the two view geometry estimation were proposed and tested constant number of samples is drawn only from  $I_k$ , while the verification is performed on the set of all data points  $U$  (inliers). Since the proportion of inliers in  $I_k$  is high, there is no need for the size of sample to be minimal. The problem has shifted from minimizing the probability of including an outlier into the sample to the problem of reduction of the influence of the noise on model parameters. The size of the sample is therefore selected to maximize the probability of drawing  $\alpha$  sample. In a final step, model parameters are polished by an iterative reweighted least squares technique.

## 3. Locally Optimized SAC Procedures

The locally optimization procedures of RANSAC and MLESAC are briefly discussed in this section.

### 3.1 LORANSAC

An enhancement of RANSAC, the locally optimized RANSAC (LO-RANSAC), is introduced by Chum and et al.(2003). The general structure of RANSAC algorithm is as follows. Repeatedly, subsets are randomly selected from the input data and fitting the model parameters.

The probability  $\eta$  of missing a set of inliers of size  $I$  within  $k$  samples falls under predefined threshold,

$$\eta = (1-P_1)^k \quad (3.1)$$

where  $P_1 = \frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I-j}{N-j} \approx \epsilon^m$ ,  $\epsilon = I/N$  and  $k = \log(\eta)/\log(1-P_1)$ .

The equation (3.1) is a selection of a single random sample not contaminated by outliers. The probability that  $k^{\text{th}}$  sample will be the best so far  $1/k$  and  $k$  sample is

$$\sum_1^k \frac{1}{x} \leq \int_1^k \frac{1}{x} dx + 1 = \log k + 1$$

The computational algorithm of LO-RANSAC procedure is as follows:

**Algorithm:**

- (i) *Standard: the standard implementation of RANSAC without any local optimization.*
- (ii) *Simple: find new model parameter (data points with error smaller than  $\theta$ ).*
- (iii) *Iterative: Reduce the threshold and iterate until the threshold is  $\theta$ .*
- (iv) *Inner RANSAC: A new sampling procedure is executed.*
- (v) *Inner RANSAC with iteration: the local optimization methods are based on the size of sample and iterative scheme observations.*

### 3.2 LOMLESAC

An enhancement of MLESAC, the locally optimized MLESAC (LO-MLESAC) is proposed by Tian et al.(2009). LOMLESAC adopts the same sample strategy and likelihood theory as the previous approach and an additional generalization model optimization step is applied to the models with the best quality.

The MLESAC, the probability density is also used for the classify inliers and outliers and it is satisfies the inequality as follows,

$$\gamma \left( \frac{1}{\sqrt{2\pi} \sigma} \exp\left(\frac{e^2_i}{2\sigma^2}\right) \right) > (1 - \gamma) \frac{1}{w} \quad (3.2)$$

The idle number of outer iterations depends on algorithms confidence level  $p(\cdot)$  and the inliers ratio  $\gamma$ :

$$I_{max} = \frac{\log(1-p(\cdot))}{\log(i-\gamma^m)} \quad (3.3)$$

The LOMLESAC algorithm can be summarized as follows.

**Algorithm:**

- (i). The minimum set  $S_m$  is sampled.
- (ii). Use  $S_m$  to estimate the fundamental matrix  $F$ .
- (iii). Calculate likelihood function.
- (iv). If the cost is minimal apply the optimal model with LO(locally optimize).
- (v). Update  $I_{max}$  using new inliers ratio.
- (vi). Repeat the above steps until the number of loops reaches  $I_{max}$ .

#### 4. Locally Optimized SSAC (LOSSAC)

Most of the sample consensus (SAC) regressions estimates associated with M-scale. The weakness of M estimation is the lack of consideration on the distribution of data and not a function of the overall data because only using the median as the weight value. The SSAC algorithm is based on residuals standard deviation to overcome the weaknesses of median. Further, the locally optimized algorithm is introduced in the SSAC estimator to speed up the process of detecting maximum of number of inliers (LOSSAC). The LOSSAC procedure is as follows:

First, estimate  $\hat{\beta} = (\min)\beta \hat{\sigma}_s (e_1, e_2, \dots, e_n)$  with determining minimum robust scale estimator  $\hat{\sigma}_s$  and satisfied the following conditions

$$\min \sum_{i=1}^n \left( \frac{y_i - \sum_{j=1}^k x_{ij}\beta}{\hat{\sigma}_s} \right) \text{ where } \hat{\sigma}_s = \sqrt{\frac{1}{nk} \sum_{i=1}^n w_i e_i^2}$$

$k=0.199$ ,  $w_i = w_\sigma(u_i) = \frac{\rho(u_i)}{u_i^2}$  and the initial estimate is

$$\hat{\sigma}_s = \frac{\text{median}|e_i - \text{median}(e_i)|}{0.6745}$$

The solution is obtained by differentiating to  $\beta$  so that,

$$\sum_{i=1}^n x_{ij} \phi(y_i - \sum_{j=0}^k x_{ij}\beta) = 0, j=0, 1, 2, 3, \dots, k. \tag{4.1}$$

$\phi$  is a function as derivative of  $\rho$ .

$$\phi(u_i) = \rho'(u_i) = \begin{cases} u_i \left[ 1 - \left( \frac{u_i}{c} \right)^2 \right]^2, & |u_i| \leq c \\ 0, & |u_i| \geq c \end{cases}$$

where  $w_i$  is an IRLS (Iterative Re-weighted Least Square) function,

$$w_i(u_i) = \begin{cases} \left[ 1 - \left( \frac{u_i}{c} \right)^2 \right]^2, & |u_i| \leq c \\ 0, & |u_i| > c \end{cases} \tag{4.2}$$

where  $u_i = \frac{e_i}{\sigma_s}$  and  $c = 1.547$ .

The computational algorithm of LOSSAC is as follows:

The structure of the SSAC algorithm is simple and powerful. Repeatedly, subsets are randomly selected from the input data and model parameters fitting the sample are computed. In a second step, the quality of the model parameter is evaluated on data set. Different cost function is to be used for evaluation.

#### Algorithm

- (i). Select a random sample of the min. ( $S_s$ )
- (ii). Estimate the model parameter  $\beta$
- (iii). Calculate the number of inliers  $I_\alpha$
- (iv). If  $\max(I_\alpha > I_i \text{ for } i < \alpha)$ , run local optimization and store the best model.

$P_\alpha = \epsilon^n$ , Symbol  $P_\alpha$  stands for the probability that an uncontaminated sample of size  $n$  is randomly selected from  $N$  data points, where  $\epsilon$  is the fraction of inliers.

The LOSSAC procedure finds more number of inliers to find near to the optimum I. These inliers are achieved via local optimization of best samples. The locally optimized step is carried out only if a new maximum in the number of inliers form the current sample has occurred. The density function of the k<sup>th</sup> sample will be the best as far as 1/k. The average number of best sample is

$$\sum_i^k \frac{1}{x} \leq \int_1^k \frac{1}{k} dx + 1 = \log k + 1 \tag{4.3}$$

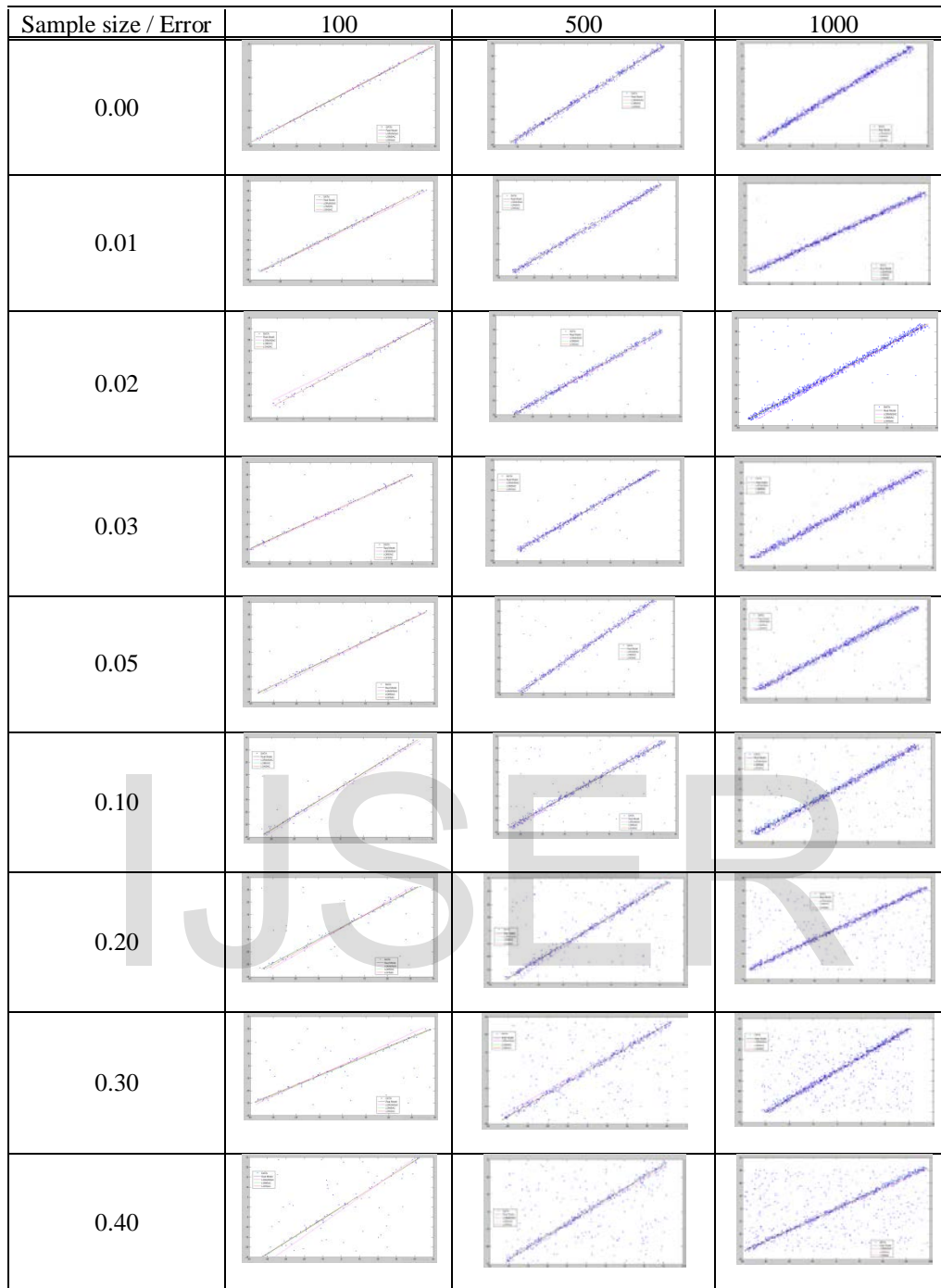
It is finite and upper bound as the number of correspondences is discrete. This theoretical bound was confirmed experimentally, the average number of local optimization over and execution of SSAC.

### 5. Experimental Results

The performance of the proposed LOSSAC has been carried out with LORANSAC and LOMLESAC family of estimators. The data were generated under real line model under varying sample sizes. The experiment was conducted to estimate the number of inliers with various levels of contaminations. The obtained results such as maximum number of inliers and robust fitting are summarized in the table 5.1 and figure 5.1.

**Table 5.1: Estimated number of inliers under LOSSAC with other procedures**

Data	n=100			n=500			n=1000		
Models →	LORANSAC	LOMLESAC	LOSSAC	LORANSAC	LOMLESAC	LOSSAC	LORANSAC	LOMLESAC	LOSSAC
Error ↓	LORANSAC	LOMLESAC	LOSSAC	LORANSAC	LOMLESAC	LOSSAC	LORANSAC	LOMLESAC	LOSSAC
0.00	100	100	100	500	500	500	1000	1000	1000
0.01	99	99	99	498	498	498	994	994	994
0.02	97	97	98	492	491	493	982	980	983
0.03	97	94	97	489	486	489	977	974	977
0.05	96	95	96	480	479	481	963	963	965
0.10	95	95	95	464	463	465	919	912	922
0.20	83	83	83	423	420	425	840	841	841
0.30	78	78	78	387	381	388	762	761	764
0.40	71	71	72	357	359	360	690	690	697



**Figure 5.1: Regression fit under LOSSAC with other procedures**

It is observed from the results, that the LOSSAC achieves the best performance among the other estimators. In the context of maximum number of inliers with various levels of contamination, the LOSSAC estimated more number of inliers compared with LORANSAC and LOMLESAC.

## 6. Conclusions

The locally optimization technique has been introduced in the SSAC procedure in order to enhance the speed and to estimate the more number of inliers, namely Locally optimized SSAC (LOSSAC) was introduced in this paper. The LOSSAC apparently achieves much result

(detected inliers) compared to LORANSAC and LOMLESAC techniques. Based on the experiments the proposed LOSSAC procedure shows more efficient and perceptual quality compared to those of reference methods. This paper concludes that, the LOSSAC algorithm not only maximizes the support of model by improving its precision, but it can also switch to a more complex model with more accurate fit.

## References

- [1]. Bui, T.D., Yu, X., and Krzyzak, A., 1994, “*Robust estimation for range image segmentation and reconstruction*”, vol.16(5), pp. 530-538.
- [2]. Chum, O., Matas, J., and Kittler, J., 2003, “*Locally Optimized RANSAC*”, proceedings of DAGM-symposium, pp. 236-243.
- [3]. Chum, O., and Matas, J., 2002, “*Randomized ransac with  $T(d,d)$  test*”, In Proceedings of the British Machine Vision Conference, volume 2, pages 448–457,.
- [4]. Lauchlan, P.Mc., and Jaenicke, A., 2000, “*Image mosaicing using sequential bundle adjustment*”, In Proc. BMVC, pp. 616– 62.
- [5]. Leonardis, A., and Bischof, H., 2000, “*Robust recognition using Eigen images*”, Computer Vision and Image Understanding: CVIU, 78(1):99–118.
- [6]. Myatt, D., Torr, P., Nasuto, S., Bishop, J., and Craddock, R., 2002, “*NAPSAC: High noise, high dimensional robust estimation - it's in the bag*”, In BMVC02, vol. 2, pp. 458–467.
- [7]. Muthukrishnan, R., and Radha, M., 2010, “*M-Estimator in Regression Models*”, Journal of Mathematics Research, Vol.2 (4), pp. 23-27.
- [8]. Muthukrishnan, R., and Radha, M., 2012, “*INAPSAC: A New Robust Inliers Identification Technique*”, Journal of Advanced Computer Science and Technology, Vol. 1(4), pp. 284-290.
- [9]. Muthukrishnan, R., and Radha, M., 2012, “*A comparative study on Robust RANSAC Technique*”, International Journal of Statistics and Analysis, vol.2(3), pp.227-232.
- [10]. Pritchett, P., and Zisserman, A., 1998, “*Wide baseline stereo matching*”, In Proc. International Conference on Computer Vision, pp. 754–760.
- [11]. Schaffalitzky, F., and Zisserman, A., 2001, “*View point invariant texture matching and wide baseline stereo*”, In Proc. 8th ICCV, Vancouver, Canada.
- [12]. Torr, P., Zisserman, A., and Maybank, S., 1998, “*Robust detection of degenerate configurations while estimating the fundamental matrix*”, CVIU, vol.71(3), pp.312–333.

- [13]. Torr,P.H.S.,1995, “*Outlier Detection and Motion Segmentation*”, PhD thesis, Department of Engineering Science, University of Oxford.
  
- [14]. Tordoff,B., and Murray,D.,2002,“*Guided sampling and consensus for motion estimation*”, In Proceeding 7<sup>th</sup> ECCV, Copenhagen, Denmark, Springer Verlag, vol.1, pp. 82–96.
  
- [15]. Torr, P.H.S., and Zisserman,A.,2000, “*MLESAC: A new robust estimator with application to estimating image geometry*”, Computer Vision and Image Understanding, vol.78, pp.138-156.

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